

**Applying Six Sigma and Statistical
Quality Control to Optimizing
Software Inspections**

ITRA

Six Sigma for Software Development

Friday, January 24, 2002

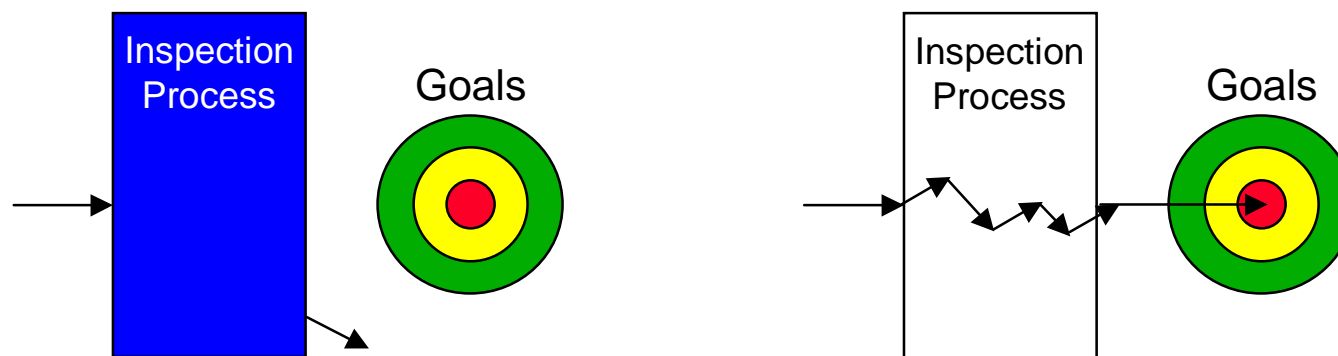
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Goals and Measurement

- An inspection process that is not *actively managed* will probably be less effective in achieving its goals. It might even be counterproductive
- “You can’t manage what you can’t measure”
- Goals should be stated measurably
- Measures should be defined



Measurements of the inspection process are key to managing the process and achieving the goals

Measurements

- **Only three basic measurements**
 - **Effort:** the effort required to prepare for, hold, and fix the defects found in, the inspection
 - **Size:** the size of the work product inspected, often measured in lines of code (LOC)
 - **Defects:** the number and type of defects, effort required to fix, point of injection and point of removal, description
- **Development effort should be proportional to size**
- **Defect density should be proportional to size**
- **Size units should be chosen so that average defect density is not “too small”**
- **Simple and economical to collect in-process with an automated tool**
- **All other metrics are derived from these three measurements**

Derived Measurements

- **Review Rate - LOC/hr**
- **Defect Density - Defects/KLOC**
- **Defect Injection Rate - Defects/hr**
- **Defect Removal Rate - Defects/hr**
- **Yield - Defects Removed/Defects Present**
- **Defect Removal Leverage - Inspection Removal Rate/Test Removal Rate**
- **Appraisal Cost of Quality – cost of all inspection activities expressed as a % of project cost**
- **Failure Cost of Quality – cost of all re-work related activities required to complete compilation and test expressed as a % of project cost**

Characterizing Variation

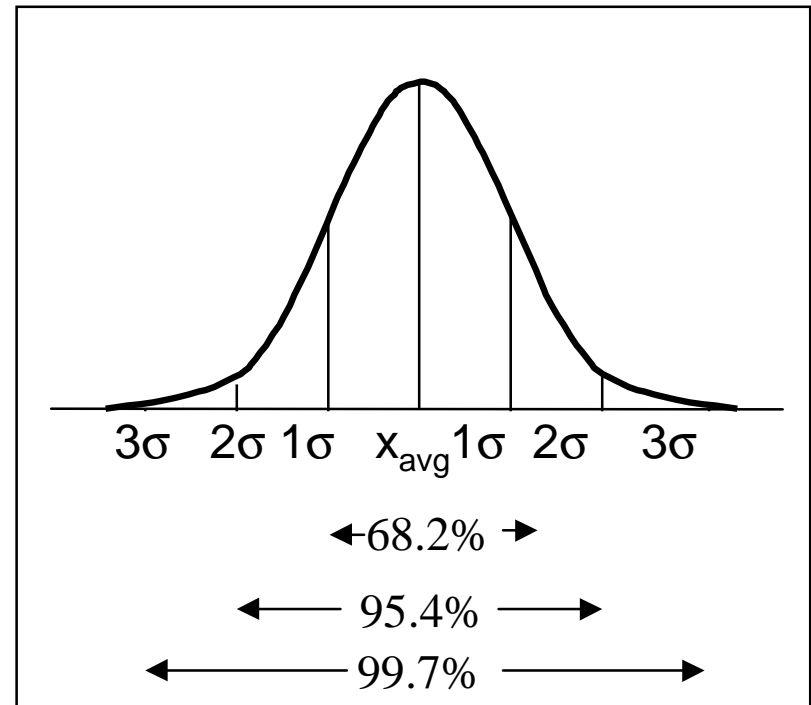
- Most data tends to follow the normal distribution or bell curve.
- The standard deviation (σ) measures variation present in the data

$$\sigma = \sqrt{\frac{1}{n-1} \sum (x - x_{avg})^2}$$

- For data that follows a normal distribution

— 99.9999975% of the data is within $\pm 6\sigma$

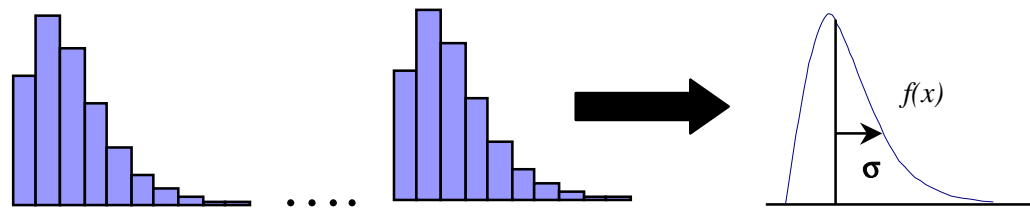
- $\pm 3\sigma$ is natural limit of random data variation produced by a process
- The empirical rule allows us to treat non-normal data as if it were normal for the purposes of statistical process control
 - 60%-75% of the data is within 1σ of the mean
 - 90%-98% of the data is within 2σ of the mean
 - 99%-100% of the data is within 3σ of mean



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Process Stability and Statistical Control

- A process exhibits statistical control when a sequence of measurements $x_1, x_2, x_3, \dots, x_n, \dots$ has a consistent and predictable amount of variation
- It is possible to model this pattern of variation with a stationary probability density function $f(x)$

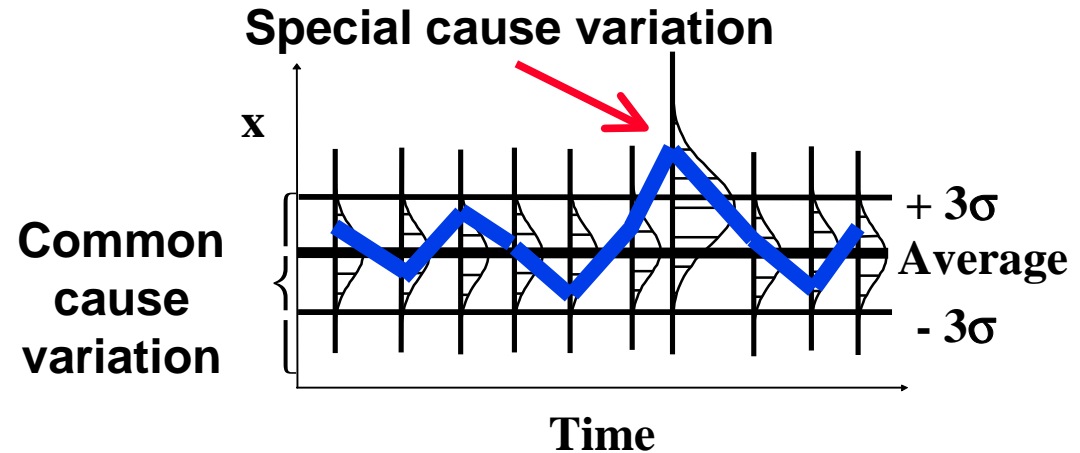


- Can make statistically valid predictions about processes that exhibit statistical control
- When the process does not exhibit statistical control, the distribution function changes over time, destroying the ability to make statistically valid predictions
- A stable well-defined process is a pre-requisite for statistical control

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Control Charts and Process Variation

- Control charts are a graphical depiction of the normal range of variation of a stable process



- Common cause variation is normal random variation in process performance
 - Don't over-react to common cause variation
 - Reduction requires a process change
- Special cause variation represents an exception to the process
 - Actions to correct special cause variation must eliminate a specific assignable cause
 - Special cause action eliminates a specific isolated event; does not necessarily involve a process change
- Don't take special cause action to deal with common cause problem

XmR Charts

- Used with continuous data (measurements)
- no assumptions about underlying distribution
- Appropriate for items that are not produced in “batches” or when it is desirable to use all available data
- two charts: X and mR (moving Range of X)
- mR_{avg} is used to estimate σ for X as well as mR

- $mR_i = |X_i - X_{i-1}|$
- X chart mean: X_{avg}
- X chart control limits: $X_{avg} \pm 2.660 mR_{avg}$
- mR chart mean: mR_{avg}
- mR chart control limit: $3.268 mR_{avg}$

\bar{X} -R and -S Charts

- Used with continuous data (measurements)
- No assumptions about underlying distribution
- Periodic subgroups (size n) used to sample data stream
 - conditions should be relatively homogeneous within subgroup
 - \bar{X}_i is subgroup average
 - R_i is subgroup range, S_i is subgroup standard deviation
 - \bar{X} chart used to identify differences between subgroups
 - R or S chart used to identify inconsistency within subgroups
 - variation within subgroups determines overall sensitivity
- \bar{X} -R
 - \bar{X} chart mean: \bar{X}_{avg}
 - \bar{X} chart control limits: $\bar{X}_{avg} \pm A_2 R_{avg}$
 - R chart mean: R_{avg}
 - R chart control limits: ($D_3 R_{avg}$, $D_4 R_{avg}$)
- \bar{X} -S ($n \geq 10$)
 - \bar{X} chart mean: \bar{X}_{avg}
 - \bar{X} chart control limits: $\bar{X}_{avg} \pm A_3 S_{avg}$
 - S chart mean: S_{avg}
 - S chart control limits: ($B_3 R_{avg}$, $B_4 R_{avg}$)
- $A_2(n)$, $A_3(n)$, $B_3(n)$, $B_4(n)$, $D_3(n)$, $D_4(n)$ are tabulated coefficients

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nP and P Charts

- **Used with discrete binomial data (number of failures)**
- **Likelihood of item's failure unaffected by failure of previous item in the sample**
- **nP charts**
 - x_i : number of failures in a sample
 - fixed sample size n
 - average fraction non-conforming p
 - Mean: np
 - Constant control limits: $np \pm 3 [np(1 - p)]^{1/2}$
- **P charts**
 - p_i : proportion of failures in a sample
 - variable sample size n_i
 - Mean: p
 - Variable control limits: $p \pm 3 [p(1 - p)/n_i]^{1/2}$
 - Control limits tighten up for larger sample sizes and relax for smaller sample sizes

C and U charts

- Used with discrete Poisson data (count of defects/sample)
- independent events (defects)
- probability proportional to area of opportunity (sample size)
- events are 'rare' (< 10% possible defects)
- C charts
 - c_i : event count
 - constant area of opportunity
 - average number of events per sample c_{avg}
 - Mean: c_{avg}
 - Constant control limits: $c_{avg} \pm 3(c_{avg})^{1/2}$
- U charts
 - u_i : event count per unit area of opportunity (defects/unit size)
 - variable area of opportunity a_i
 - Mean: u_{avg}
 - Variable control limits: $u_{avg} \pm 3(u_{avg})^{1/2}$
 - Control limits tighten up for larger sample sizes and relax for smaller sample sizes

Detecting Assignable Causes

- **X is out of control whenever**
 - a single point x_i falls outside the three sigma control limits CL_x
 - at least two out of three successive x_i 's fall on the same side of, and more than two sigma units away from, the central line
 - at least four out of five successive x_i 's fall on the same side of, and more than one sigma unit away from, the central line
 - at least 8 successive x_i 's fall on the same side of the central line
- **R is out of control when**
 - 8 or more successive r_i 's fall on same side of median
 - or 12 or more successive r_i 's fall on same side of mR
- **A trend is any upward or downward movement of 5 or more consecutive points**
- **Use of control charts to quantify normal variation and to identify the presence of assignable causes is called Statistical Process Control (SPC)**

***Never attempt to interpret the X chart
when the mR chart is out of control !***

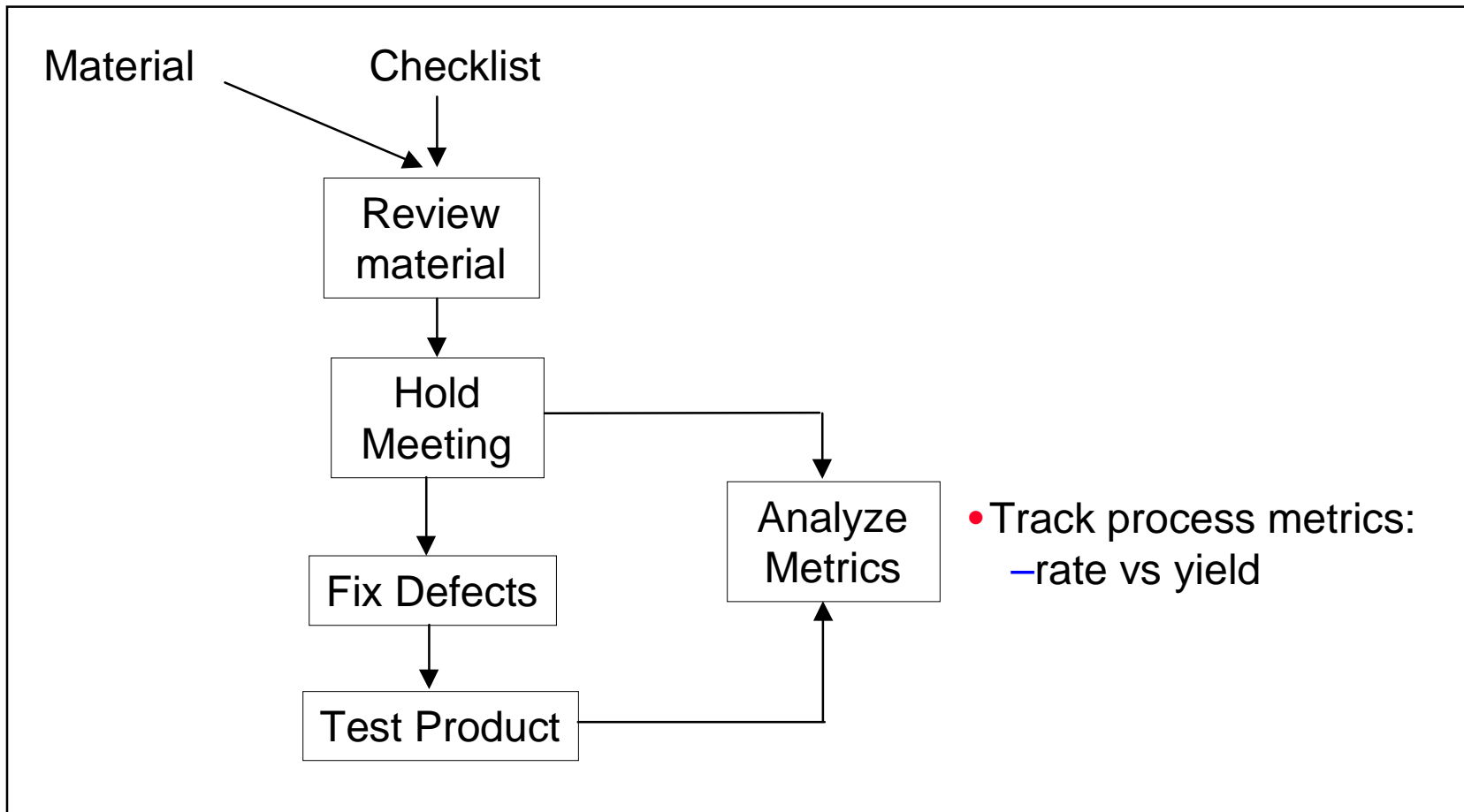
Continuous vs Discrete Data

- For discrete data with a narrow range of values, e.g. counts of rare events, then XmR charts will exhibit quantization problems and one of the discrete charts will be more appropriate
- If discrete data looks approximately continuous (high average count), it is possible to still use XmR charts
 - Quantization becomes significant when the standard deviation is less than a single size unit
 - For count data that has a Poisson Distribution, the standard deviation is proportion to the square root of the average, so that if the average defect count is ≥ 2 , it is reasonable to treat the data as continuous
 - So XmR charts can almost always be used
- If all the assumptions behind the discrete charts are met, they will outperform XmR charts, however they are not as general and may mislead if used incorrectly

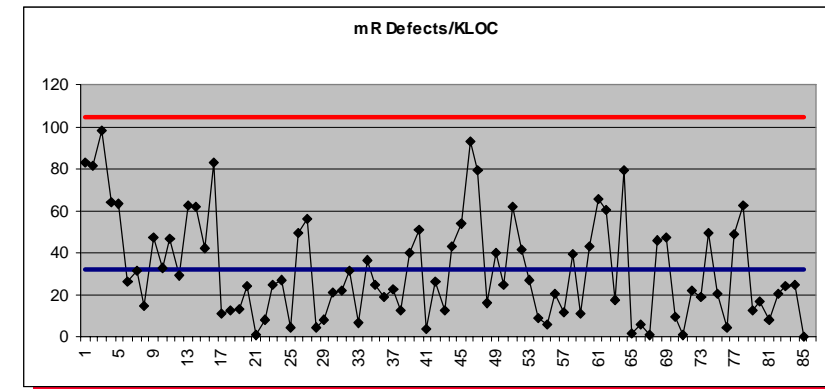
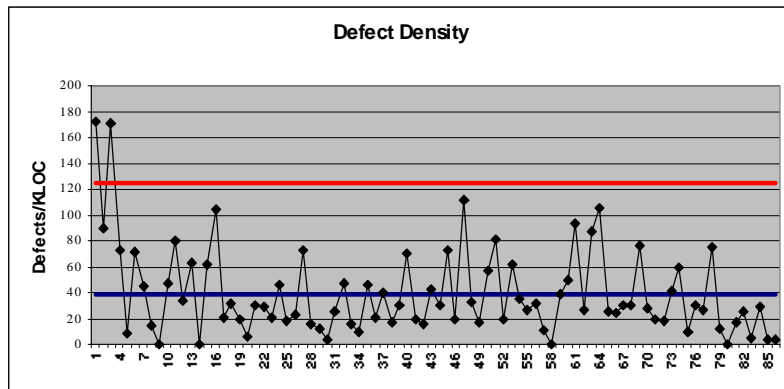
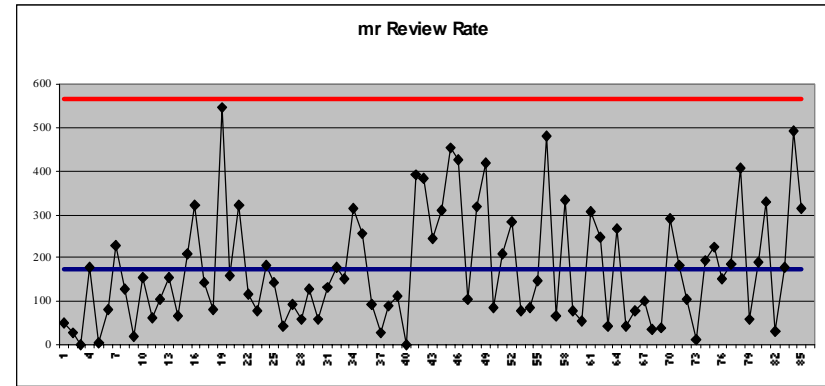
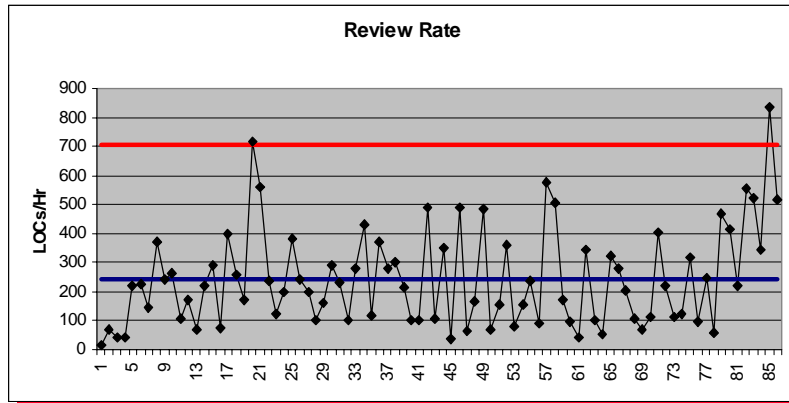
Using Control Charts with Software Metrics

- **20 to 30 data points are typically used to calculate control limits.**
 - **Fewer data points can produce limits that are too wide and run the risk of missing a signal. Out of control points will still be out of control when the limits are tighter, so it is best to simply revise limits when more data becomes available rather than to wait**
- **Size and time are continuous variables, as are metrics derived from them like productivity, COQ, residual estimating errors, etc. Distributions are not normal. Sample size is one since every unit is generally different - use individual charts**
- **Defects are discrete**
- **Defects/unit size and defect rates may look continuous for large enough products and appropriate size units (typically 100 defects/KLOC)**

Open Loop Inspection Process - Tracking



Open Loop Process XmR Charts



- Average review rate 244 LOCs/Hr
- Average defect density 39 Defects/KLOC
- Average removal rate 6/Hr

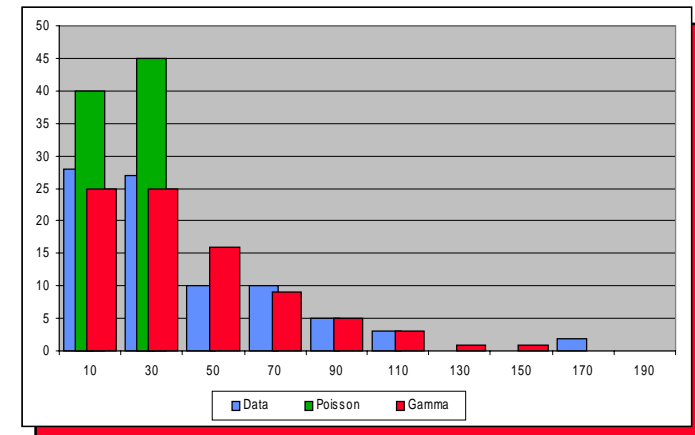
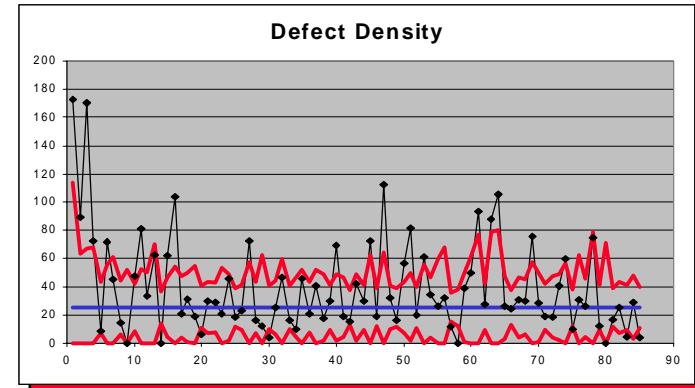
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What about U-Charts for Defects?

- Wouldn't it be better to use a U-Chart for Defect Density?
- The variable control limits are much tighter and with the u-chart, it is possible for a point to go out of control at the lower limit
- But a whole lot of points that looked in control before now seem to be out of control - what happened?
- The empirical distribution function doesn't look much like a Poisson, in fact it is more like a Gamma or Exponential
 - long tail causes “out of control” points
- U-chart underestimates normal variation because process doesn't follow Poisson Distribution!



A Control System Viewpoint

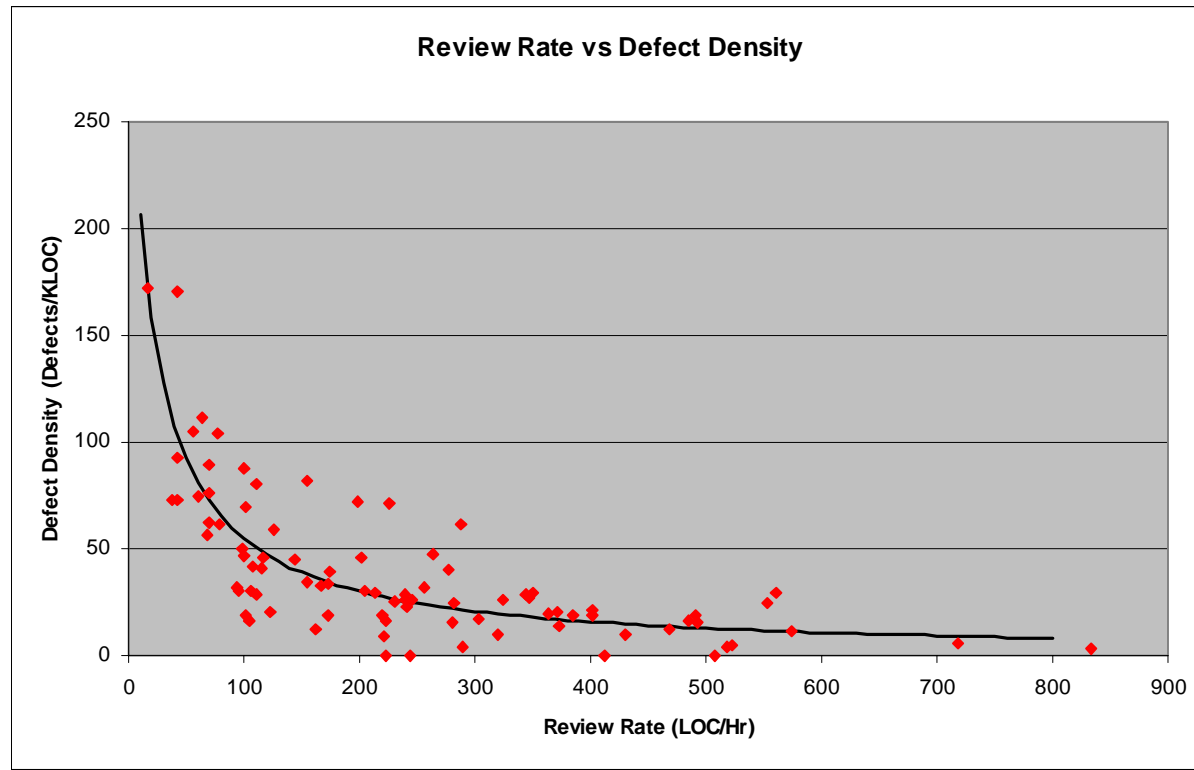
- The outputs of a process, y , are usually a function, f , of a set of control variables, x , and include a process noise component ε :

$$y = f(x) + \varepsilon$$

- The y 's are not directly controllable, but they can be controlled by the directly controllable x 's.
 - Statistical measurements are necessary to avoid re-acting to the noise ε
- Ideally we would like software inspection process that acts like a responsive, “closed loop” control system driving the x 's to planned values and through their relationship to the y 's, achieving overall product goals

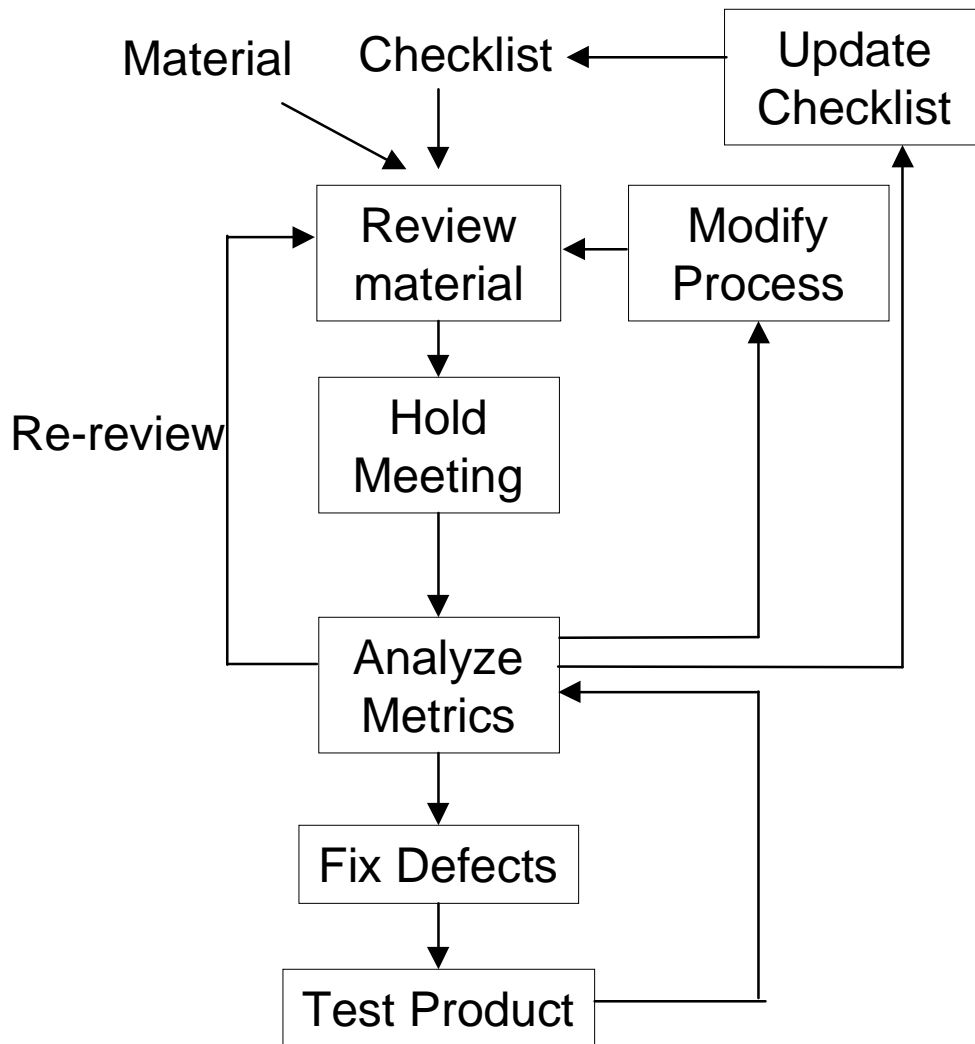
Our experience has shown that review rate is the x that drives the inspection yield

Correlation Analysis



- To evaluate review rate for suitability as a control variable use correlation analysis
- $r^2 = 0.67$ – moderately good fit by hyperbola
- Chart suggests targeting review rate in the 100 – 200 LOCs hour range

Closed Loop Inspection Process



Update Checklist

- Remove questions that are not catching defects.
- Add questions to catch defects that are leaking out to test.

Modify Process

- Modify review rate
- Vary size of material reviewed
- Include test cases

Analyze Metrics

- Process metrics:
 - Rate vs Yield
- Product metrics:
 - Compare yields to quality plan
 - Re-review of products that fall outside quality thresholds
 - Buggiest products list

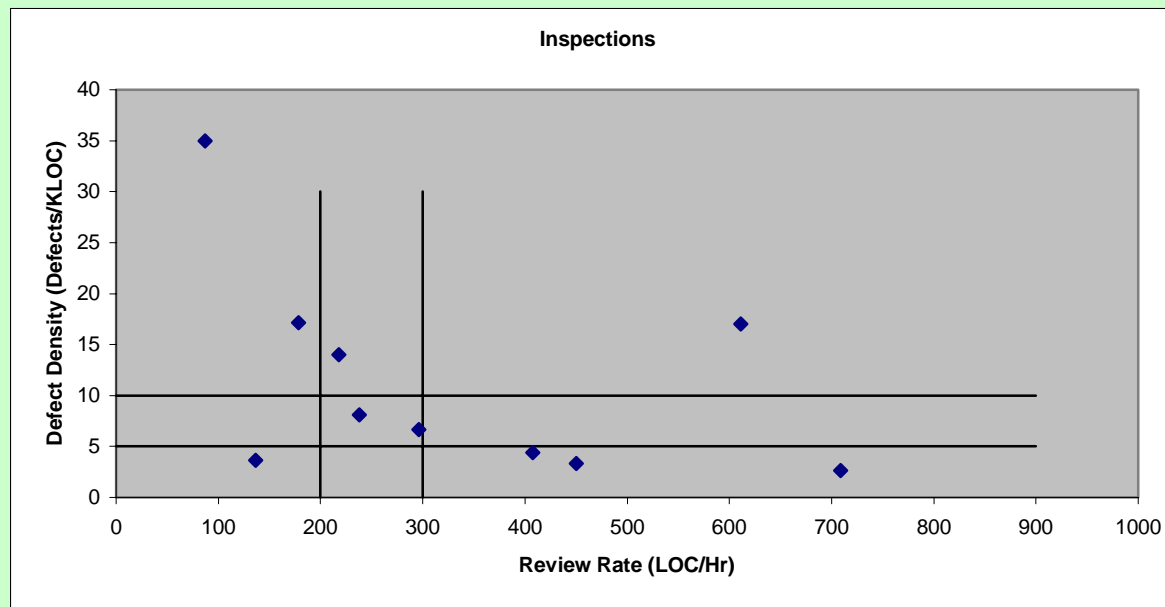
Inspection Performance Assessment

Slow Review Rate & Many Defects

Is the product really buggy?
Was the review really effective?
Was the review cost efficient?

Fast Review Rate & Many Defects => Buggy Product

The product is buggy.
Return to author for rework
Ask someone else to rewrite



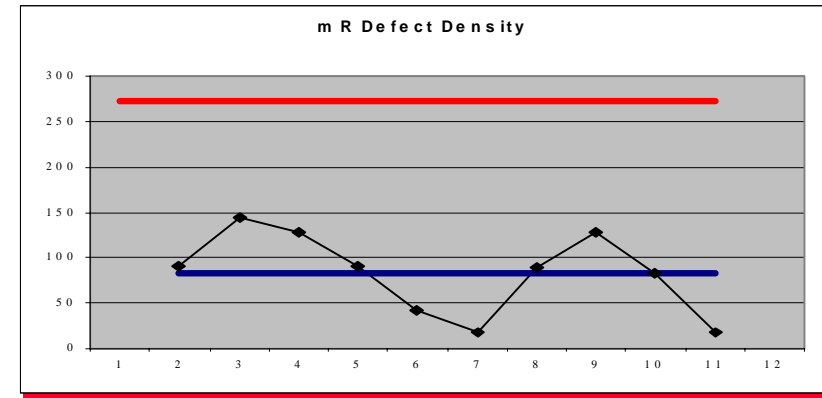
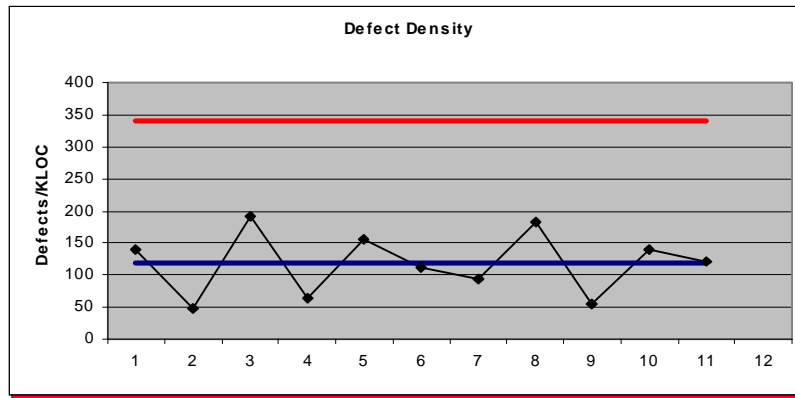
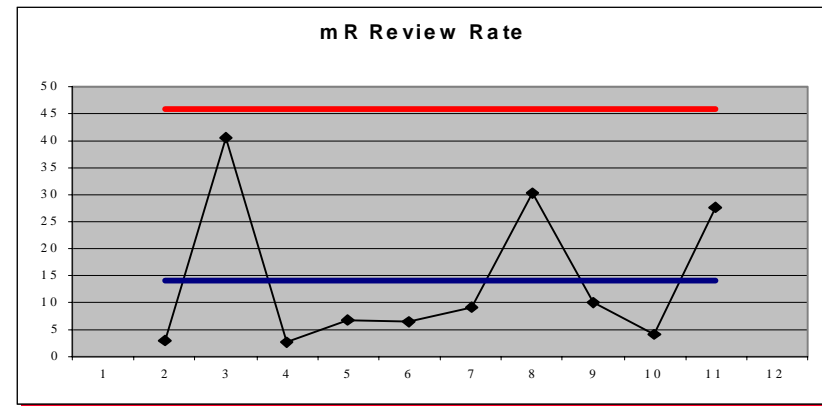
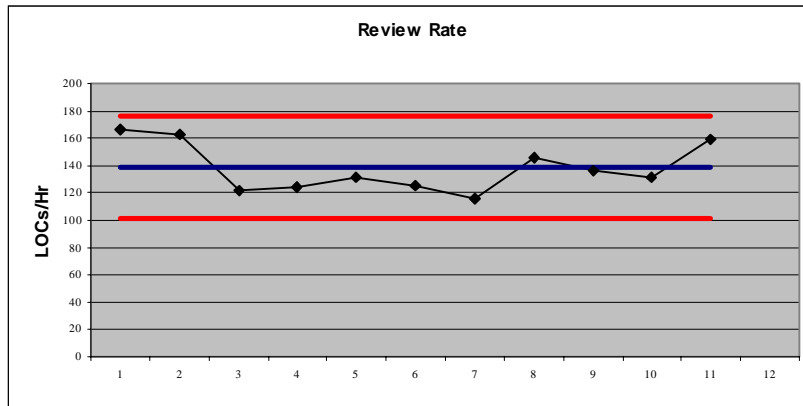
Slow Review Rate & Few Defects

Is the product really good?
Was the review really ineffective?
Was the review cost efficient?

Fast Review Rate & Few Defects => Poor Review

Is the product really good?
Re-review at a slower rate
Make sure reviewers are using the checklist

Closed Loop XmR Charts



- Targeting rate yielded major decrease in variation
- Closed loop process achieved significant improvements
 - Average Review Rate 138 LOCs/hr
 - Average Defect Density 118 Defects/KLOC
 - Average Defect Removal Rate 15/hr

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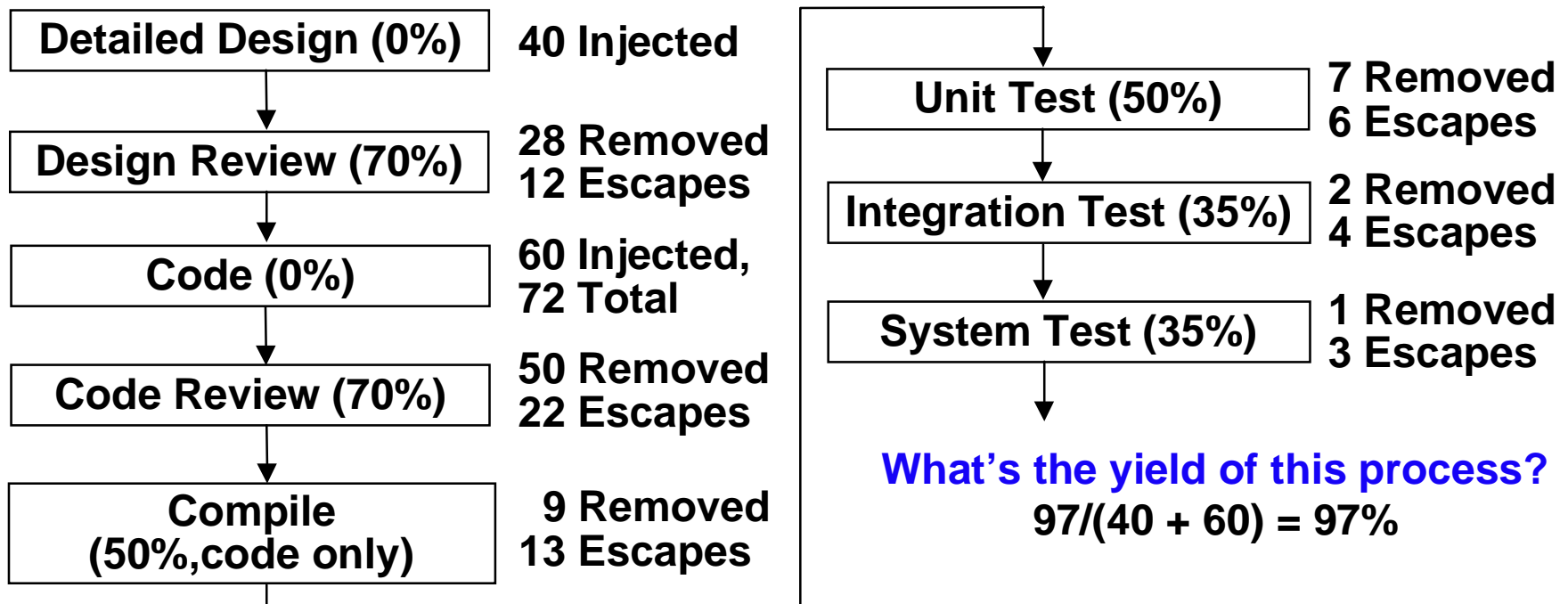
Optimization Strategy

- **Personal reviews performed prior to team inspections**
 - Remove all the errors the author can detect at the lowest possible inspection cost
 - Checklist derived from author's own list of compilation and test defects flags high risk areas where author has a history of making mistakes
- **Frequent short team inspections**
 - Checklists focus on interface and requirements related issues that can't easily be found in the personal review
 - Small teams that include the internal "customers" for the product
 - Focus on a few hundred lines of code at a time
- **Periodic Defect Prevention meetings provided the development team with an opportunity to review their data and define approaches to detect defects earlier or prevent or prevent them entirely**
- **Defect prone products "pulled" from integration and test and re-inspected**

Goal: Minimize review cost while maximizing yield

Yields and Quality Planning and Management

- Inspection process can be characterized by its yield
- Historical yields permit planning the number of defects that will be removed
- Manage to the plan by taking corrective action when actual values diverge from plan



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Calculating Return on Investment - 1

- Costs can be directly measured
 - training, tools, performing the inspections
- The dominant costs are the inspection prep and the meeting time
- Savings require estimating the difference in cost between finding a defect in review and finding it later in the process

	Defects leaked from prev phase	New Defects Injected	Phase Yield	Defects Contained	Defects Leaked	Defect Removal Cost	Total Removal Cost (hrs)
Design	0.0	40	0%	0.0	40.0	n/a	0.00
Design Bench Check	40.0	0	0%	0.0	40.0	10 mins	0.00
Design Inspection	40.0	0	0%	0.0	40.0	30 mins	0.00
Code	40.0	60	0%	0.0	100.0	n/a	0.00
Code Bench Check	100.0	0	0%	0.0	100.0	5 mins	0.00
Compile	100.0	0	50%	50.0	50.0	1 min	0.83
Code Inspection	50.0	0	0%	0.0	50.0	15 mins	0.00
Unit Test	50.0	0	50%	25.0	25.0	15 mins	6.25
Integration Test	25.0	0	35%	8.8	16.3	18 hrs	157
System Test	16.3	0	35%	5.7	10.6	18 hrs	102
CUSTOMER	10.6						267

- Without inspections, the cost of defect removal is 267 hrs per KLOC

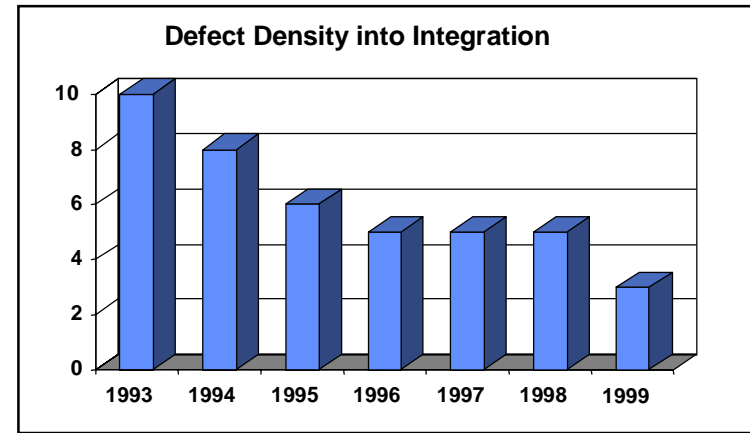
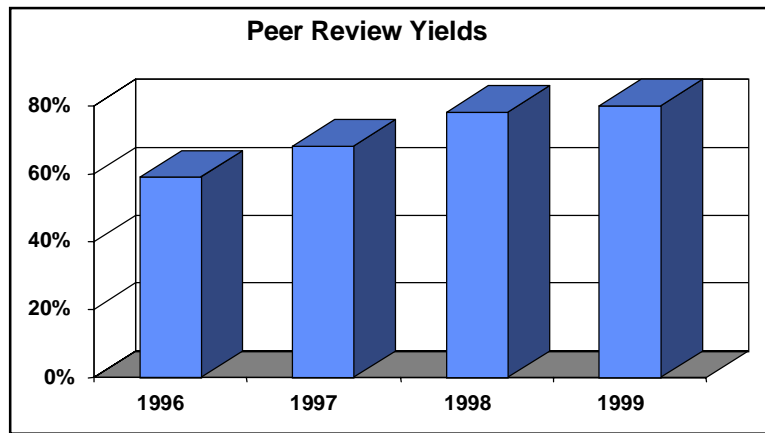
Calculating Return on Investment - 2

	Defects leaked from prev phase	New Defects Injected	Phase Yield	Defects Contained	Defects Leaked	Defect Removal Cost	Total Removal Cost (hrs)
Design	0.0	40	0%	0.0	40.0	n/a	0.00
Design Bench Check	40.0	0	50%	20.0	20.0	10 mins	3.33
Design Inspection	20.0	0	50%	10.0	10.0	30 mins	5.00
Code	10.0	60	0%	0.0	70.0	n/a	0.00
Code Bench Check	70.0	0	70%	49.0	21.0	5 mins	4.08
Compile	21.0	0	50%	10.5	10.5	1 min	0.18
Code Inspection	10.5	0	60%	6.3	4.2	15 mins	1.58
Unit Test	4.2	0	50%	2.1	2.1	15 mins	0.53
Integration Test	2.1	0	35%	0.7	1.4	18 hrs	13.23
System Test	1.4	0	35%	0.5	0.9	18 hrs	8.60
CUSTOMER	0.9						37

- With inspections, the cost of defect removal drops to 37 hours, a savings of $230 = 267 - 37$ hours
- The cost of holding the inspections is about 40 hours (at 200 LOC/hr), so the net savings is 190 hours

Results

- Over a period of 5 years, we gradually implemented the strategies described
- As Peer Review yields increased from 60% to 80% and we introduced personal reviews, defects into integration were reduced from 10/KLOC to 3/KLOC
- At the same time, cost of performing peer reviews decreased by 40% as we reduced the size of the inspection teams



The organization realized a net improvement of 190 hrs / KLOC!

Glossary of Terms

CMM®	Capability Maturity Model
COQ	Cost Of Quality
EV	Earned Value
KLOC	Thousand Lines Of Code
LOC	Lines Of Code
ROI	Return On Analysis
SEI	Software Engineering Institute
SPC	Statistical Process Control
SPI	Software Process Improvement

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References

- A more detailed introduction on using Six Sigma techniques to measure and control process variation was provided earlier at this conference in: [*Six Sigma and Software Process Improvement*](#)
- An explanation of how to use Six Sigma techniques in conjunction with Personal Software Process and Team Software Process is being presented later at this conference on *Wed, Nov 20 at 3:45* in: [*Integrating PSP, TSP and Six Sigma*](#)
- For additional information see our web site or to answer any questions contact:

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